Quasicrystal Cooling

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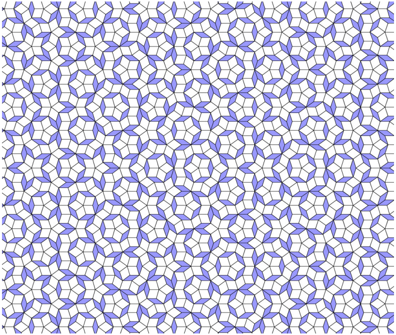
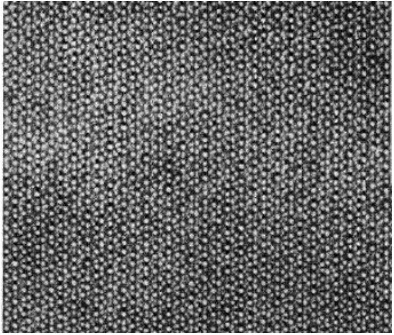
CNRS& Univ. Paris 13

TransTile
June 2016

Quasicrystals

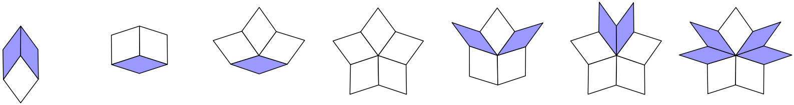
Penrose quasiperiodic tiling *vs* $\mathrm{Al_{71}Mn_{24}Fe_5}$ quasicrystal:





Structure & Growth

The Penrose tilings are characterized by a set of *allowed patterns*:



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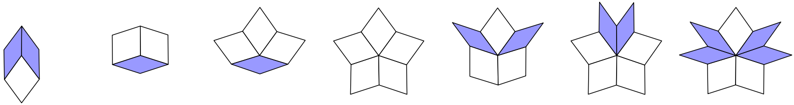
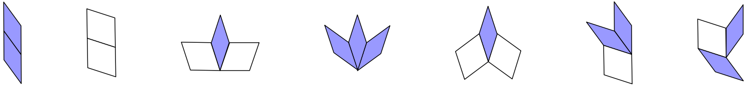


Equivalently, they are characterized by a set of *forbidden patterns*:



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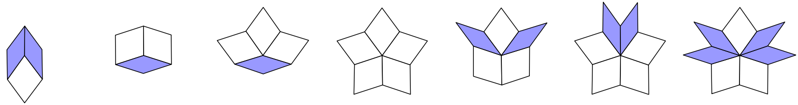

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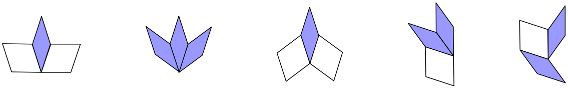

Question 1

Which (aperiodic) tilings can be characterized by finite patterns?

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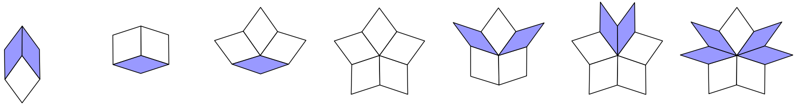

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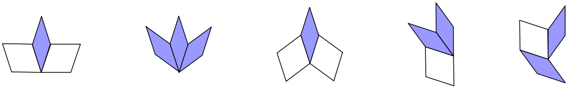


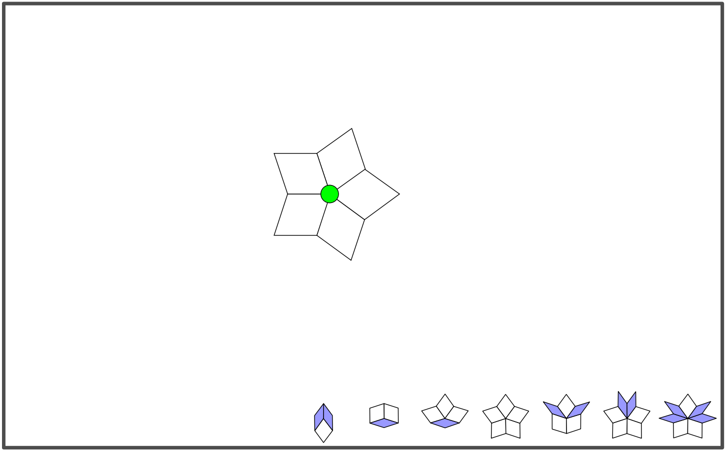
Question 1

Which (aperiodic) tilings can be characterized by finite patterns?

Question 2

How to "grow" a tiling from finite patterns which characterize it?

Deceptions and correcting flips



Let us try to grow tile by tile a Penrose tiling.

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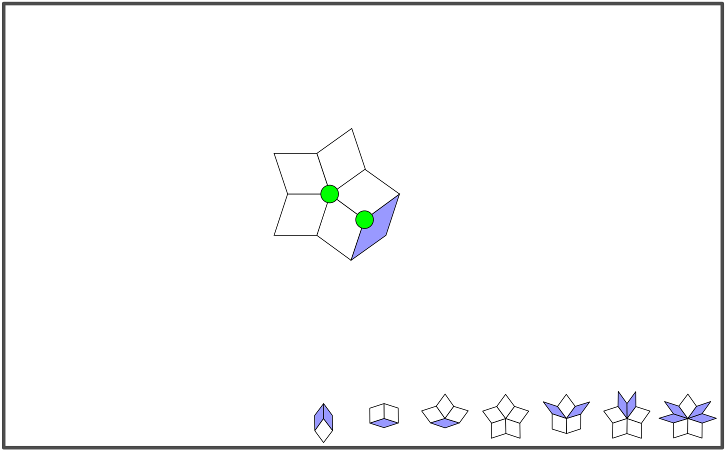
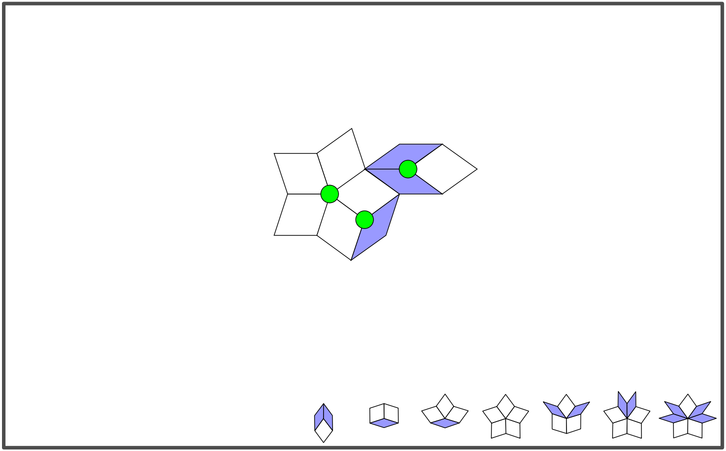


We simply need to form only allowed patterns.

Deceptions and correcting flips



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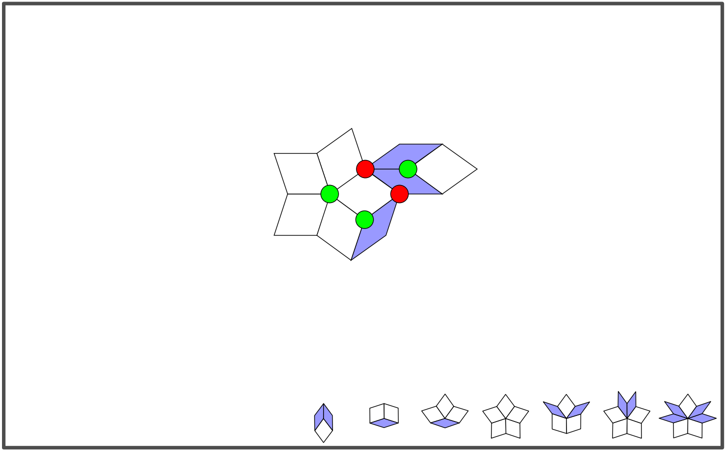
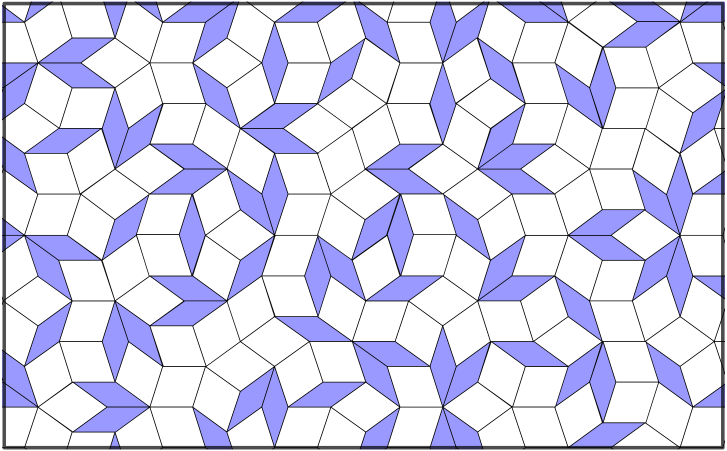


Oops! We created *deceptions*! Usual business with aperiodicity. . .

Deceptions and correcting flips



Let us forget allowed patterns and just grow a “random” tiling.

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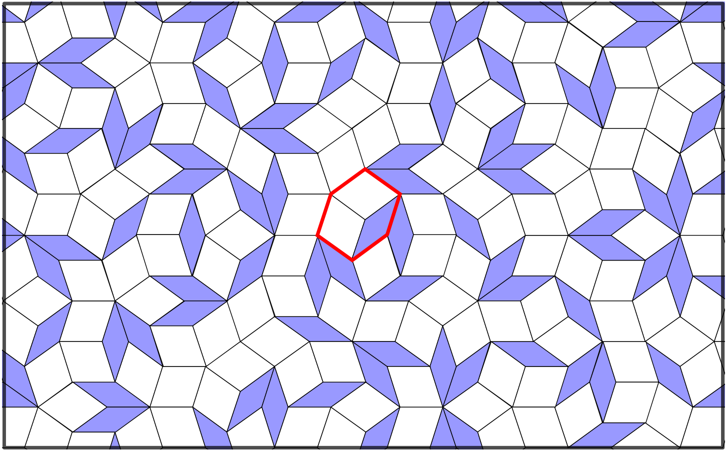
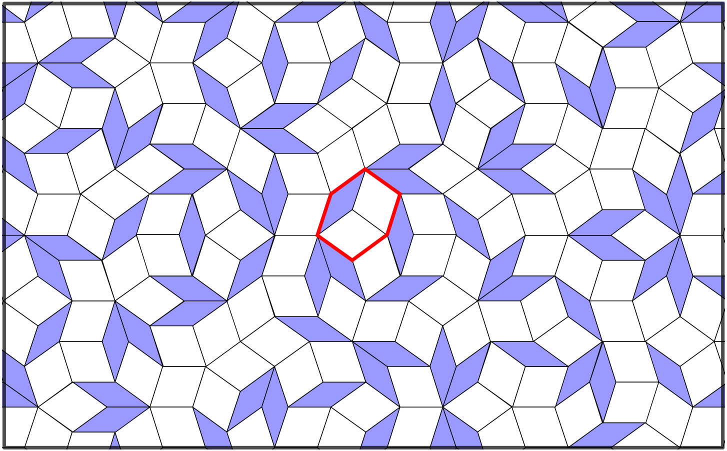


We shall try to correct it afterward by local moves called *flips*.

Deceptions and correcting flips



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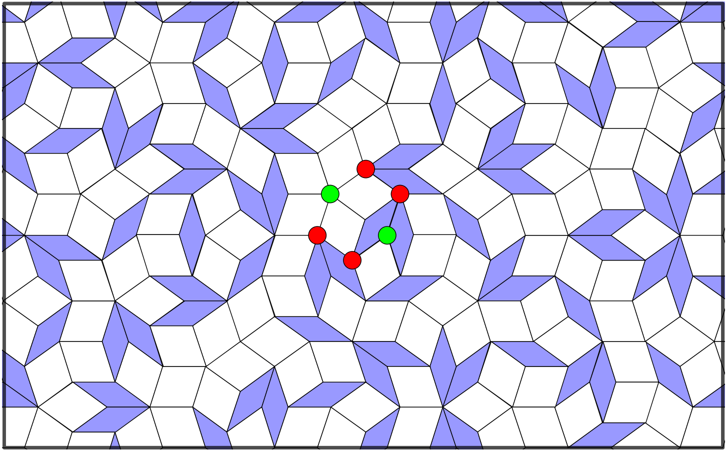

Forbid *e.g.* flips which decrease the number of allowed patterns.

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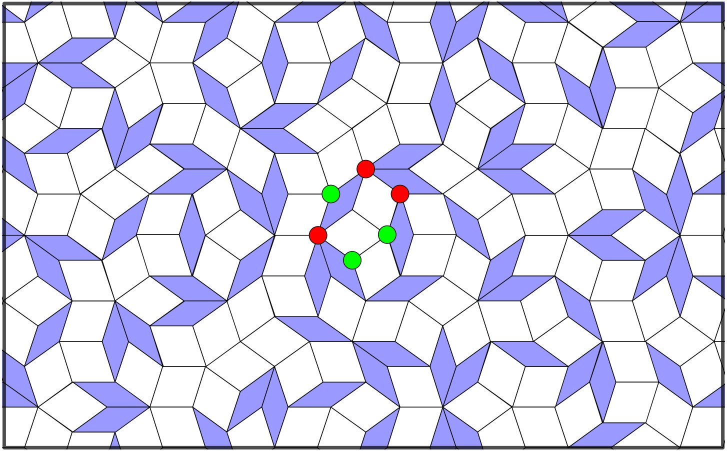

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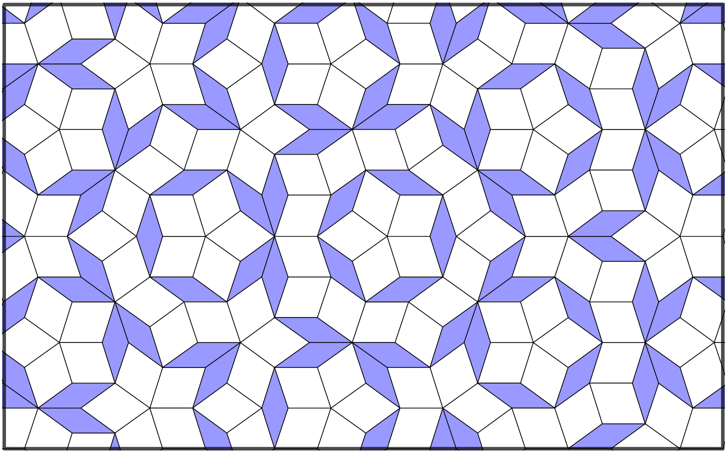

Do we eventually get only allowed patterns? When?

Formally

Consider the tilings by rhombi of a finite simply connected region.

Fix a finite set of forbidden finite patterns.
Define the *energy* E of a tiling as its number of forbidden patterns.

Consider the discrete time Markov chain at temperature T:

- pick a vertex *u.a.r.*
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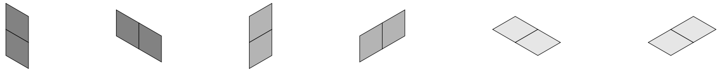
The chain is known to be ergodic at $T > 0$ (Kenyon'93).
Is it *rapidly mixing*? For any T or with some *phase transition*?

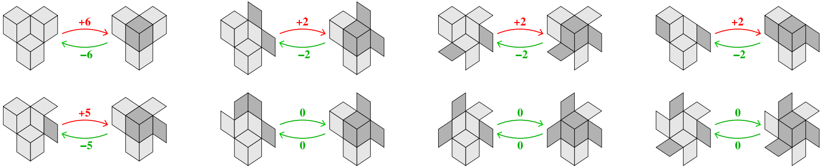
Example 1: dimers at infinite temperature

At $T = \infty$, flips are all performed with probability 1.
Forbidden patterns (energy) do not play any role!

This is nothing but the usual random sampling of dimer tilings.
Ask Dana or Benoît for the mixing time, Cédric for the limit shape!

Example 2: alternating dimers at zero temperature





Forbid adjacent identical tiles. Forbid flip increasing the energy.

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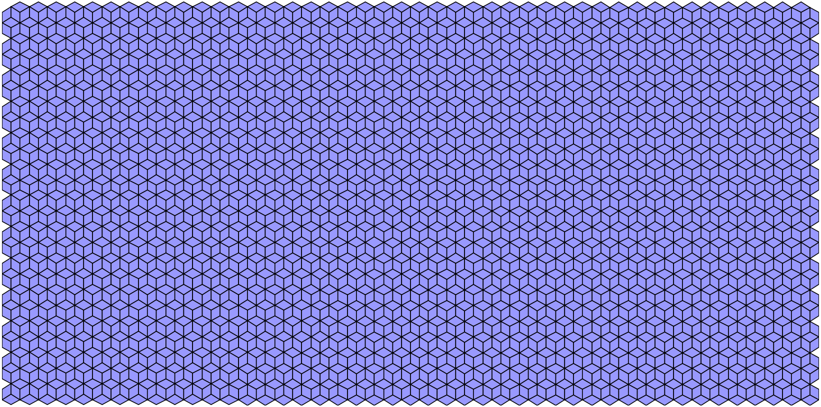

It characterize the tilings whose dimers alternate (*groundstate*).

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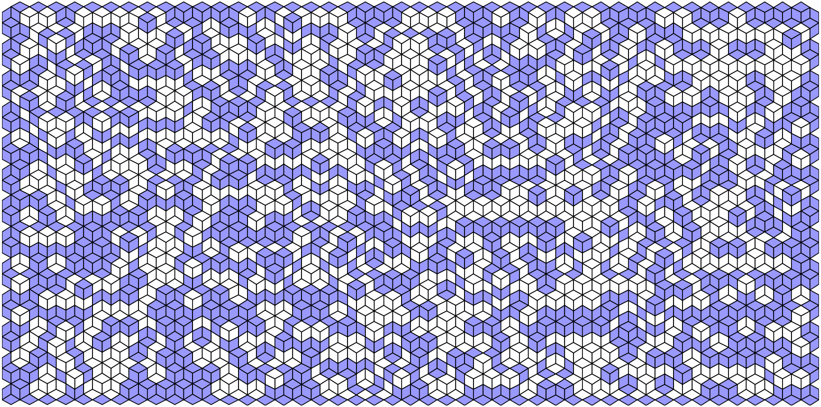

The chain is not ergodic but rapidly reaches a groundstate.

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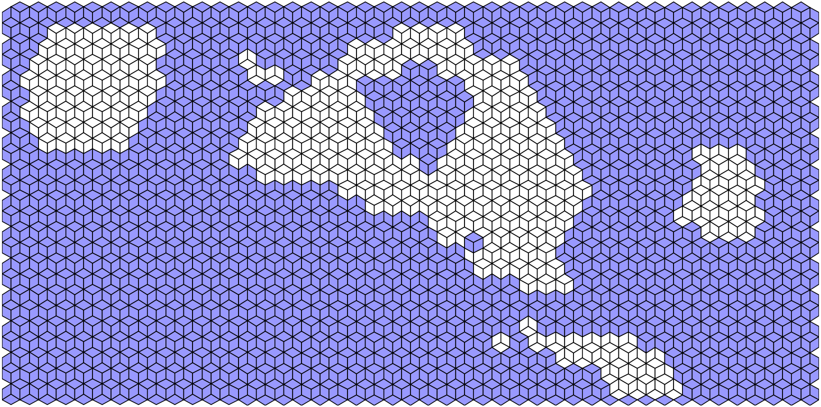

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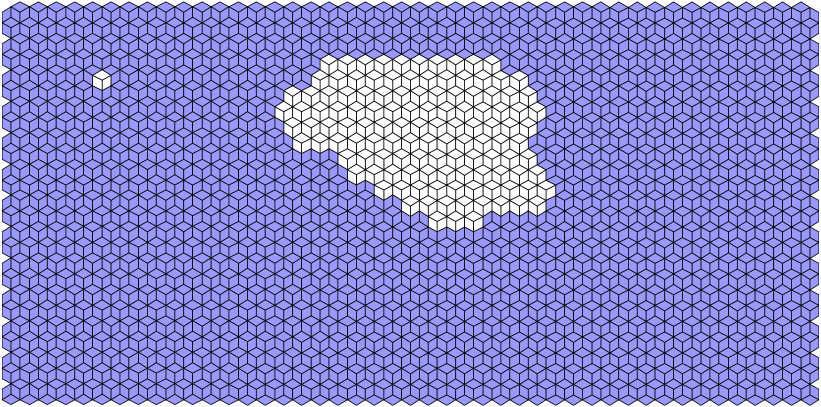

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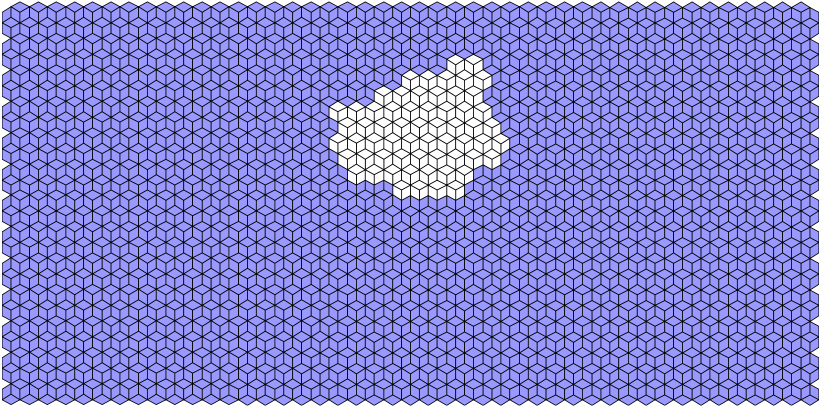

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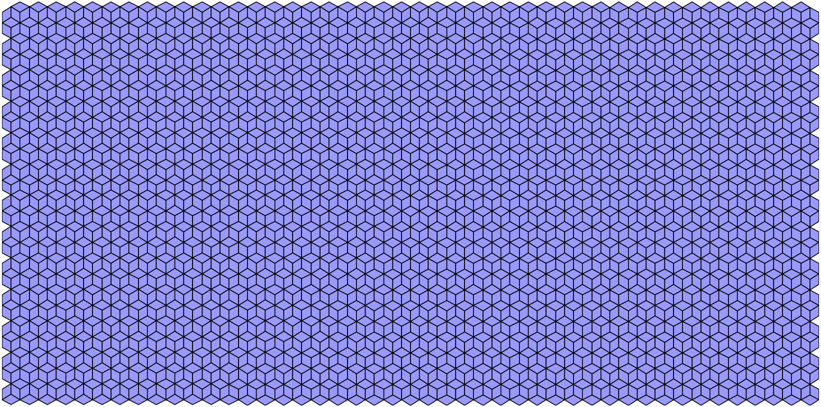

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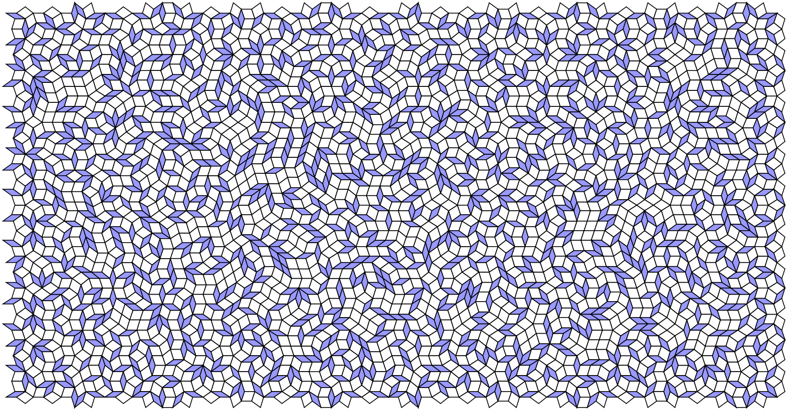

A similar process on Penrose tiling seems to rapidly converge.

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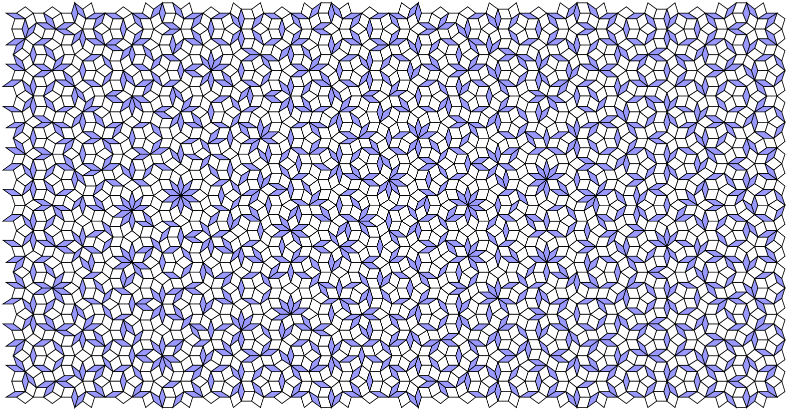

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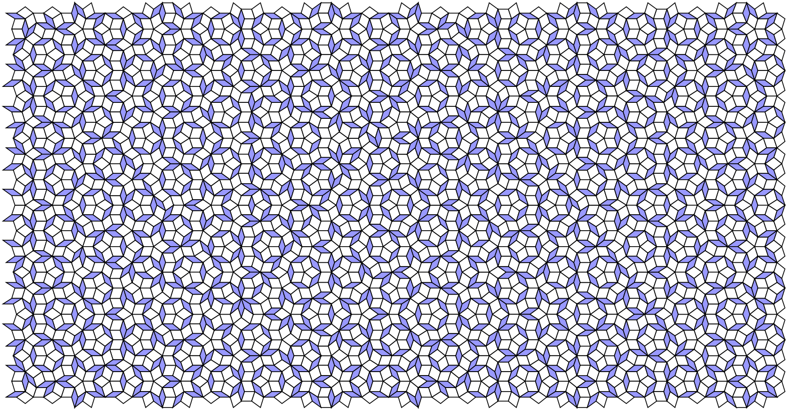

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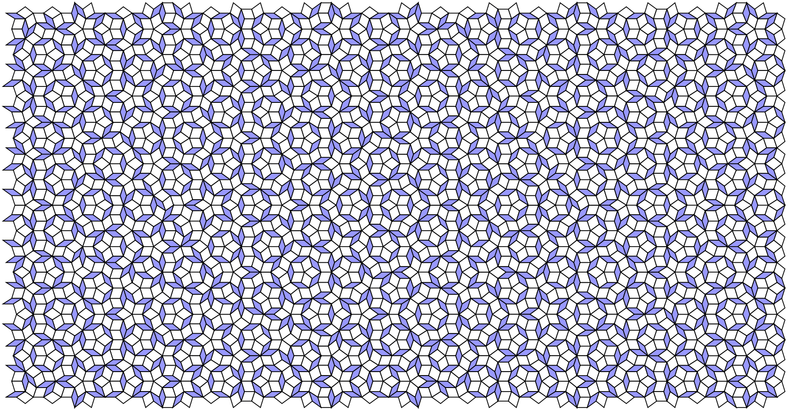

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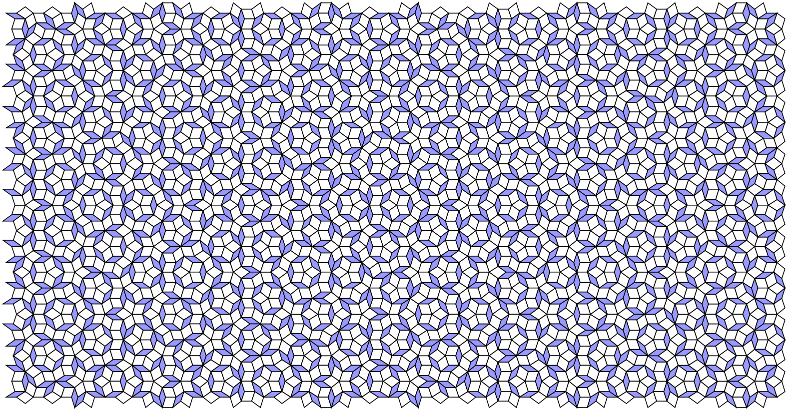

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